

Chapter 1

Introduction

Motivation

In every field of human life there are processes that cannot be described exactly (by an algorithm). For example, how fast does a web page respond? when does the bus come? how many cars are on the parking lot at 8.55 am?

By observation of these processes or by experiments we can detect patterns of behavior, such as: “ usually, the first week of semester the campus network is slow”, “by 8.50 am the parking lot at the Design building usually is full”.

Our goal is to analyze these patterns further:

1.1 Basic Probability

Real World	Mathematical World
Observation, experiment with unknown outcome	Random experiment
list of all possible outcomes	sample space Ω (read: Omega)
individual outcome	elementary event A , $A \in \Omega$ (read: A is an element of Ω)
a collection of individual outcomes	event A , $A \subset \Omega$ (read: A is a subset of Ω)
assignment of the likelihood or chance of a possible outcome	probability of an event A , $P(A)$.

1.1.1 Examples for sample spaces

1. I attempt to sign on to AOL from my home - to do so successfully the local phone number must be working and AOL's network must be working.

$$\Omega = \{ (phone\ up, network\ up), (phone\ up, network\ down), (phone\ down, network\ up), (phone\ down, network\ down) \}$$

2. Online I attempt to access a web page and record the time required to receive and display it (in seconds).

$$\Omega = (0, \infty) \text{ seconds}$$

3. on a network there are two possible routes a message can take to a destination - in order for a message to get to the recipient, one of the routes and the recipient's computer must be up.

Ω_1 in tabular form:

route 1	route 2	recipient's computer
up	up	up
up	up	down
up	down	up
up	down	down
down	up	up
down	up	down
down	down	up
down	down	down

or, alternatively: $\Omega_2 = \{ \text{successful transmission, no transmission} \}$

Summary 1.1.1

- Sample spaces can be finite, countable infinite or uncountable infinite.
- There is no such thing as *THE* sample space for a problem. The complexity of Ω can vary, many are possible for a given problem.

1.1.2 Examples for events

With the same examples as before, we can define events in the following way:

1. $A = \text{fail to log on}$
 $B = \text{AOL network down}$

then A is a subset of Ω and can be written as a set of elementary events:

$$A = \{ (\text{phone up, network down}), (\text{phone down, network up}), (\text{phone down, network down}) \}$$

Similarly:

$$B = \{ (\text{phone up, network down}), (\text{phone down, network down}) \}$$

2. $C = \text{at least 10 s are required, } C = [10, \infty)$.
3. $D = \text{message gets through}$
 D with first sample space: $D = \{ (U, U, U), (U, D, U), (D, U, U) \}$

Once we begin to talk about events in terms of sets, we need to know the standard notation and basic rules for computation:

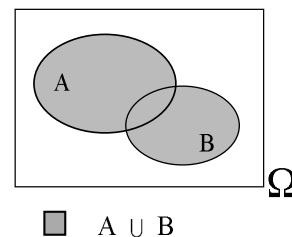
1.2 Basic Notation of Sets

For the definitions throughout this section assume that A and B are two events.

Definition 1.2.1 (Union)

$A \cup B$ is the event consisting of all outcomes **in A or in B or in both**

read: A or B



Example 1.2.1

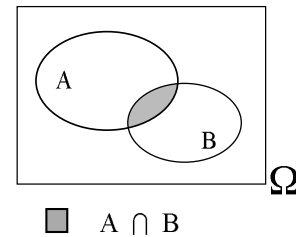
2. time required to retrieve and display a particular web page. Let A, B, C and D be events: $A = [100, 200)$, $B = [150, \infty)$, $C = [200, \infty)$ and $D = [50, 75]$.

Then $A \cup B = [100, \infty)$ and $A \cup C = [100, \infty)$ and $A \cup D = [50, 75] \cup [100, 200]$

Definition 1.2.2 (Intersection)

$A \cap B$ is the event consisting of all outcomes **simultaneously in A and in B** .

read: A and B

**Example 1.2.2**

2. Let A, B, C and D be defined as above.

Then

$$A \cap B = [100, 200) \cap [150, \infty) = [150, 200)$$

$$A \cap D = [100, 200) \cap [50, 75] = \emptyset$$

3. Let A be the event “fail to log on” and $B =$ “network down”.

Then

$$A \cap B = \{(phone\ up, network\ down), (phone\ down, network\ down)\} = B$$

B is a subset of A .

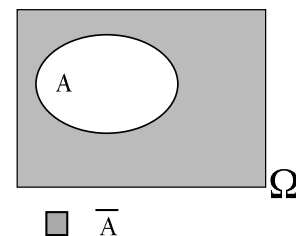
Definition 1.2.3 (Empty Set)

\emptyset is the the set with no outcomes

Definition 1.2.4 (Complement)

\bar{A} is the event consisting of all outcomes **not in A** .

read: not A

**Example 1.2.3**

3. message example

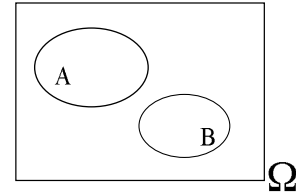
Let D be the event that a message gets through.

$$\bar{D} = \{ (D,D,U), (D,U,D), (U,D,D), (D,D,D) \}.$$

Definition 1.2.5 (disjoint sets)

Two events A and B are called *mutually exclusive* or *disjoint*, if their intersection is empty:

$$A \cap B = \emptyset$$



[A, B are disjoint